1.

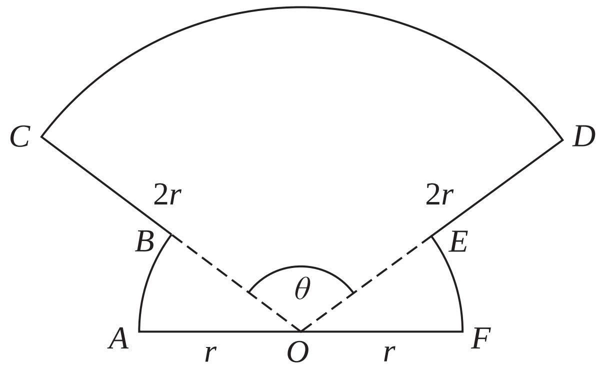


Figure 1

The shape *OABCDEFO* shown in Figure 1 is a design for a logo.

In the design
- *OAB* is a sector of a circle centre *O* and radius *r*
- sector *OFE* is congruent to sector *OAB*
- *ODC* is a sector of a circle centre *O* and radius $2r$
- *AOF* is a straight line

Given that the size of angle *COD* is θ radians,

(a) write down, in terms of θ, the size of angle *AOB*

(1)

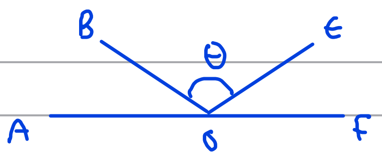
(b) Show that the area of the logo is

$$\frac{1}{2}r^2(3\theta + \pi)$$

(2)

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π.

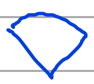
(2)

a)


$\angle AOF = \pi$ and $\angle AOB = \angle FOE$

so $\angle AOB = \frac{\pi - \theta}{2}$ 1

$= 180° = \pi^c$

b) Area $= 2 \times$  $+$ 

Area of a sector $= \frac{1}{2}\theta r^2$

θ in radians

$= 2 \times \left(\frac{1}{2}r^2\left(\frac{\pi - \theta}{2}\right)\right) + \frac{1}{2}\theta(2r)^2$ 1

$= \frac{1}{2}r^2\pi - \frac{1}{2}r^2\theta + 2r^2\theta = \frac{3}{2}r^2\theta + \frac{1}{2}r^2\pi = \frac{1}{2}r^2(3\theta + \pi)$ 1

expanded brackets

c) arc length $= r\theta$, θ in radians

arc length $CD = 2r\theta$

arc length AB and EF $= \left(\frac{\pi - \theta}{2}\right)r$

Perimeter $= 4r + 2r\left(\frac{\pi - \theta}{2}\right) + 2r\theta$ (1)

$= 4r + r\theta + r\pi$

$= r(4 + \pi + \theta)$ (1)

2.

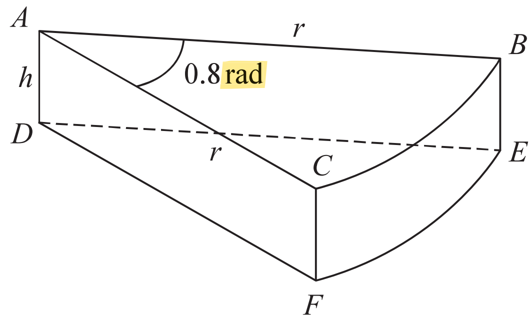


Figure 5

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face ABC is a sector of a circle with radius r cm and centre A
- angle $BAC = 0.8$ radians
- faces ABC and DEF are congruent
- edges AD, CF and BE are perpendicular to faces ABC and DEF
- edges AD, CF and BE have length h cm

Given that the volume of the toy is $240\,\text{cm}^3$

(a) show that the surface area of the toy, $S\,\text{cm}^2$, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

(2)

(a) $\underbrace{\frac{1}{2} \times 0.8 \times r^2}_{\text{area of sector}} \times \underbrace{h}_{\text{height}} = \underbrace{240}_{\text{volume}}$ 1

$\frac{1}{2}\theta r^2$ = area of sector (when θ is in radians)

$0.4r^2h = 240$ $\div 0.4$

$r^2 h = 600$ $\div r^2$

$h = \frac{600}{r^2}$ 1

total surface area = 2 × area of sector face + 2 × area of sector length + area of arc

$$S = 2\left(\tfrac{1}{2}\theta r^2\right) + 2(rh) + (r\theta \times h)$$

$$S = 0.8r^2 + 2rh + 0.8rh$$

$h = \frac{600}{r^2}$

$$S = 0.8r^2 + 2r\left(\frac{600}{r^2}\right) + 0.8r\left(\frac{600}{r^2}\right) \quad (1)$$

$$S = 0.8r^2 + \frac{1200}{r} + \frac{480}{r}$$

$$S = 0.8r^2 + \frac{1680}{r} \quad (1)$$

(b) $S = 0.8r^2 + 1680r^{-1}$ $\qquad \frac{1}{x} = x^{-1}$

$$\frac{dS}{dr} = 0.8 \times 2r^{2-1} + (-1) \times 1680r^{-1-1}$$

$$= 1.6r - 1680r^{-2} \quad (2)$$

$$0 = 1.6r - \frac{1680}{r^2} \quad (1)$$ $\leftarrow \frac{dS}{dr} = 0$ at stationary point

$$1.6r = \frac{1680}{r^2}$$

$\times r^2$

$$1.6r^3 = 1680$$

$\div 1.6$

$$r^3 = 1050$$

$\sqrt[3]{\ }$

$$r = \sqrt[3]{1050}$$

$$r = 10.16 \quad (1)$$

(c) $\frac{dS}{dr} = 1.6r - 1680r^{-2}$

$\frac{d^2S}{dr^2} = 1.6 \times 1r^{1-1} - (-2) \times 1680r^{-2-1}$

$= 1.6 + 3360r^{-3}$

$= 1.6 + \frac{3360}{r^3}$

when $r = 10.16$: ← from part (b), stationary point at $r = 10.16$

$1.6 + \frac{3360}{(10.16)^3} = 4.80$ (1)

$\frac{d^2S}{dr^2} > 0$ when $r = 10.16$ therefore this is a minimum value of S. (1)

3.

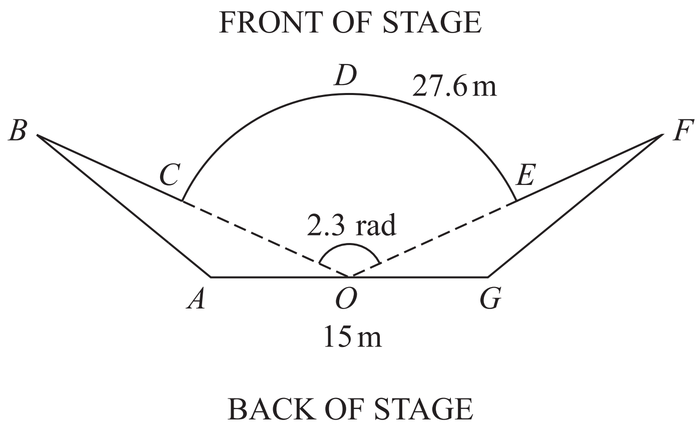


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles ABO and GFO joined to a sector $OCDEO$ of a circle, centre O, where

- angle $COE = 2.3$ radians
- arc length $CDE = 27.6$ m
- AOG is a straight line of length 15 m

(a) Show that $OC = 12$ m.

(2)

(b) Show that the size of angle AOB is 0.421 radians correct to 3 decimal places.

(2)

Given that the total length of the front of the stage, $BCDEF$, is 35 m,

(c) find the total area of the stage, giving your answer to the nearest square metre.

(6)

a) arc length $CDE = r\theta = 27.6$ 1

$\theta = 2.3 \therefore \quad r = \frac{27.6}{2.3} = 12$ 1

$r = OC \therefore OC = 12$

b) $\angle AOG = \pi$ and $\angle AOB = \angle GOF$ so

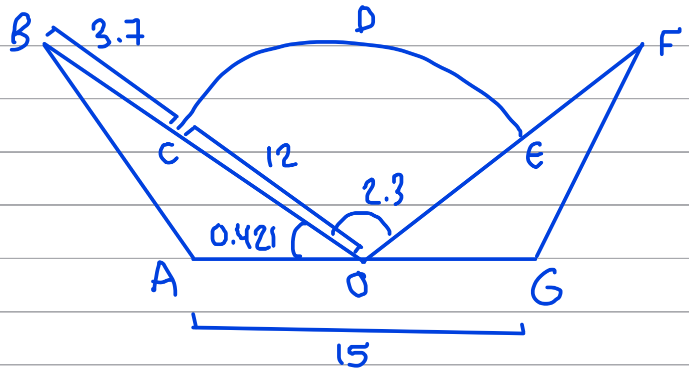
$\angle AOB = \frac{\pi - 2.3}{2} = 0.421$ rad (3dp) 1 1

c) $BC = EF$

$\therefore\ 2BC + CDE = 35$

$2BC + 27.6 = 35$

$BC = \dfrac{35 - 27.6}{2} = 3.7 \quad \therefore OB = 3.7 + 12 = 15.7$ (1)



Area $\triangle AOB = \frac{1}{2} \times \frac{15}{2} \times (3.7 + 12) \times \sin 0.421$ (1) using $\frac{1}{2}ab\sin C$

$= 24.04\ldots$

$= 24.1$ (3sf) m^2

Area $OCDE = \frac{1}{2} \times 12^2 \times 2.3 = 165.6\ m^2$ (1) (1) using $\frac{1}{2}r^2\theta$

Total area $= 165.6 + 2 \times 24.1$ (1)

$= 213.69\ldots$

$= 214\ m^2$ (nearest square metre) (1)